6.895 Randomness and Computation	March 6, 2006
Homework 2	
Lecturer: Ronitt Rubinfeld	*Due Date: March 20, 2006*

Homework guidelines: Same as for homework 1.

1. Say that f_1, f_2, f_3, mapping from group G to H, are *linear consistent* if there exists a linear function $\phi : G \to H$ (that is $\forall x, y \in G, \phi(x) + \phi(y) = \phi(x + y)$) and $a_1, a_2, a_3 \in H$ such that $a_1 + a_2 = a_3$ and $f_i(x) = \phi(x) + a_i$ for all $x \in G$. A natural choice for a test of linear consistency is to verify that

$$Pr_{x,y \in_r G}[f_1(x) + f_2(y) \neq f_3(x + y)] \leq \delta$$

for some small enough choice of δ.

- Show that f, g, h are linear-consistent iff for every $x, y \in G$ $f(x) + g(y) = h(x + y)$.
- Let $G = \{+1, -1\}^n$ and $H = \{+1, -1\}$. First note that since $a_i \in \{+1, -1\}$, then linear consistent f_i must be linear functions or "negations" of linear functions. We refer to the union of linear functions and the negations of linear functions as the *affine functions*. In class we expressed the minimum distance of f to a linear function. Express the minimum distance of a function f to an affine function.
- Show that if f_1, f_2, f_3 satisfy the above test, then for each $i \in \{1, 2, 3\}$, there is an affine function g_i such that $Pr_{x \in_r G}[f_i(x) \neq g_i(x)] \leq \delta$.
- (Extra credit) Show that there are linear consistent functions g_1, g_2, g_3 such that for $i \in \{1, 2, 3\}$, $Pr_{x \in_r G}[f_i(x) \neq g_i(x)] \leq \frac{1}{2} - \frac{2\gamma}{3}$ where $\gamma = \frac{1}{2} - \delta$.

2. Show that for any monotone function $f : \{+1, -1\} \to \{+1, -1\}$, the influence of the i^{th} variable is equal to the value of the Fourier coefficient of $\{i\}$, that is $\inf_i(f) = \hat{f}(\{i\})$.

3. Show that the majority function $f(x) = \text{sign}(\sum_i x_i)$ maximizes the total influence among n-variable monotone functions mapping $\{+1, -1\}^n$ to $\{+1, -1\}$, for n odd.

4. Let an (n, m, k)-concentrator be a bipartite graph with n left nodes and m right nodes, such that every subset of k or fewer left nodes is connected to at least the same number of right nodes. Show that for each n, there are $(n, 2n/3, n/2)$-concentrators that have $O(n)$ edges. (You may use the approximation that $\binom{n}{k} < (\frac{ne}{k})^k$).