6.895 Randomness and Computation	May 1, 2006
Homework 5	
Lecturer: Ronitt Rubinfeld	*Due Date: May 10, 2006*

1. Prove that for every function Ext : $\{0,1\}^n \to \{0,1\}$, there is an SV-source X with parameter δ such that $Pr[\text{Ext}(X) = 1] \leq \delta$ or $Pr[\text{Ext}(X) = 1] \geq 1 - \delta$.

2. First prove a seemingly strange claim about extractors that take semi-random bits but get no truly random bits, and then use it to show something interesting about extractors that take both semi-random and truly random bits. (As defined in class, if the Ext function has only one parameter, then it refers to the former type of extractor, otherwise it refers to the latter type).

   (a) Show the following: For any n and any $k < n$ and any flat k-source X, if an extractor Ext : $\{0,1\}^n \to \{0,1\}^m$ with $m = k - 2\log\frac{1}{\epsilon} - O(1)$ is chosen at random from the functions mapping $\{0,1\}^n$ to $\{0,1\}^m$, then with probability at least $1 - 2^{-\Omega(2^k \epsilon^2)}$, Ext$(X)$ is ϵ-close to U_m (the uniform distribution on m bits) with respect to statistical distance.

   (b) Then show that: For any n and $k < n$, and any $\epsilon > 0$, there exists a (k, ϵ)-extractor Ext : $\{0,1\}^n \times \{0,1\}^d \to \{0,1\}^m$ with $m = k + d - 2\log\frac{1}{\epsilon} - O(1)$ and $d = \log(n - k) + 2\log\frac{1}{\epsilon} + O(1)$.

3. (optional) Give the most intuitive explanation you can of the deterministic logspace algorithm for connectivity, and why it can be implemented in logspace.